Colin Blake
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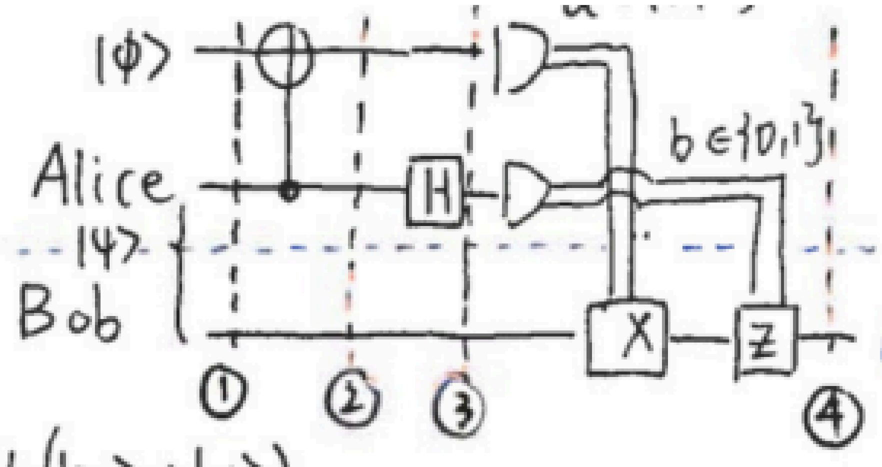
QUANTUM COMPUTING IS JUST GRAPHS!

Young Researchers' Seminar – LIGM, Institut Gaspard Monge 26 November 2025



$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle,\ \alpha, \beta \in \mathbb{C}$

$|\phi\rangle^{(1)} = \frac{1}{2\sqrt{2}}(|00\rangle + |11\rangle)(\alpha|0\rangle + \beta|1\rangle) + \frac{1}{2\sqrt{2}}(|00\rangle - |11\rangle)(\alpha|0\rangle - \beta|1\rangle) + \frac{1}{2\sqrt{2}}(|01\rangle + |10\rangle)(\alpha|1\rangle + \beta|0\rangle) + \frac{1}{2\sqrt{2}}(|01\rangle - |10\rangle)(\alpha|1\rangle - \beta|0\rangle)$

Time slice ① $|\phi\rangle|\psi\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\beta}{\sqrt{2}}|100\rangle + \frac{\beta}{\sqrt{2}}|111\rangle$ (1)

Time slice ② $(CNOT_{21} \otimes I)|\phi\rangle|\psi\rangle = \frac{1}{2\sqrt{2}}(|00\rangle + |01\rangle)(\alpha|0\rangle + \beta|1\rangle) + \frac{1}{2\sqrt{2}}(|00\rangle - |01\rangle)(\alpha|0\rangle - \beta|1\rangle) + \frac{1}{2\sqrt{2}}(|11\rangle + |10\rangle)(\alpha|1\rangle + \beta|0\rangle) + \frac{1}{2\sqrt{2}}(|11\rangle - |10\rangle)(\alpha|1\rangle - \beta|0\rangle)$

$= \frac{1}{2}|0\rangle|+\rangle(\alpha|0\rangle + \beta|1\rangle) + \frac{1}{2}|0\rangle|-\rangle(\alpha|0\rangle - \beta|1\rangle) + \frac{1}{2}|1\rangle|+\rangle(\alpha|1\rangle + \beta|0\rangle) - \frac{1}{2}|1\rangle|-\rangle(\alpha|1\rangle - \beta|0\rangle) = |\delta\rangle$

Time slice ③ $(I \otimes H \otimes I)|\delta\rangle = \frac{1}{2}|0\rangle|0\rangle(\alpha|0\rangle + \beta|1\rangle) + \frac{1}{2}|0\rangle|1\rangle(\alpha|0\rangle - \beta|1\rangle + \frac{1}{2}|1\rangle|0\rangle(\alpha|1\rangle + \beta|0\rangle)\ \frac{1}{2}|1\rangle|1\rangle(\alpha|1\rangle - \beta|0$

Time slice ④: Case 1: $\langle 00|$ Outcome $\{a=0, b=0\}$ $Z^0X^0(\alpha|0\rangle + \beta|1\rangle) = |\phi\rangle$.

Case 2: $\langle 01|$ Outcome $\{a=0, b=1\}$ $Z^1X^0(\alpha|0\rangle - \beta|1\rangle) = \alpha|0\rangle + \beta|1\rangle = |\phi\rangle$

Case 3: $\langle 10|$ Outcom $\{a=1, b=0\}$ $Z^0X^1(\alpha|1\rangle + \beta|0\rangle) = \alpha|0\rangle + \beta|1\rangle = |\phi\rangle$

Case 4: $\langle 11|$ Outcome $\{a=1, b=1\}$ $Z^1X^1(\alpha|1\rangle - \beta|0\rangle) = \alpha|0\rangle + \beta|1\rangle = |\phi\rangle$

Source: Lia Yeh's work

WHY "QUANTUM COMPUTING IS JUST GRAPHS"?

Problem: Quantum computing looks scary

Idea : Forget matrices; think **graphs+rewrite rules :** keep only the effect of a program,

Toolkit: The ZX-calculus = a Graphical language

Today's goal:

- make quantum computing feel **accessible** and **visual**,
- show how **graph rewriting** can **simplify** circuits,
- hint at **connections with combinatorics, logic, and algorithms**

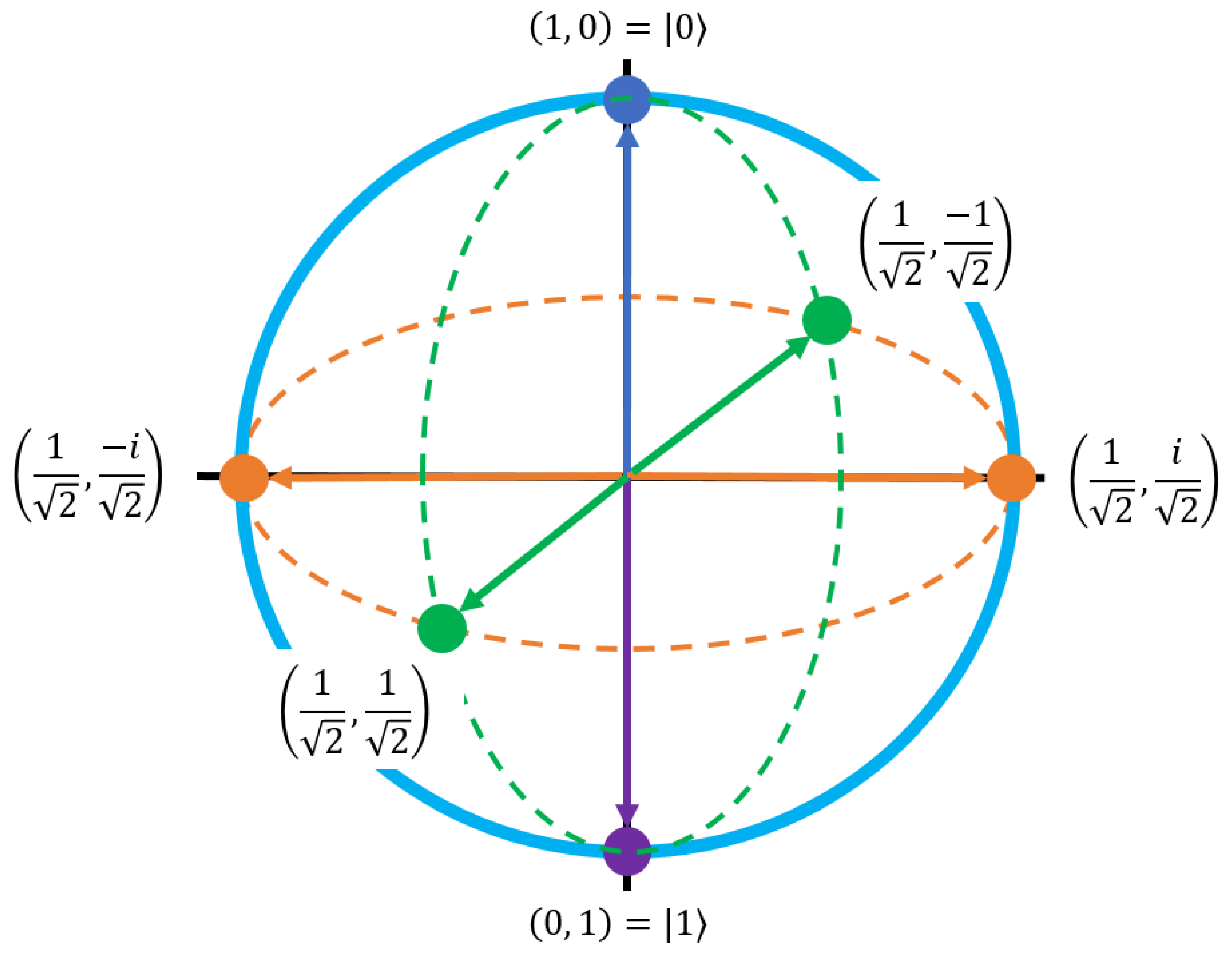
AGENDA

1. Key concepts in quantum computing
2. From circuits to graph rewriting
3. ZX-calculus rewrite rules
4. Examples of simplifications
5. Demo: ZXLive in action
6. Open problems & my current research
7. Connections with LIGM & conclusion

KEY CONCEPTS IN QUANTUM COMPUTING (1/4)

QUBITS AND SUPERPOSITION

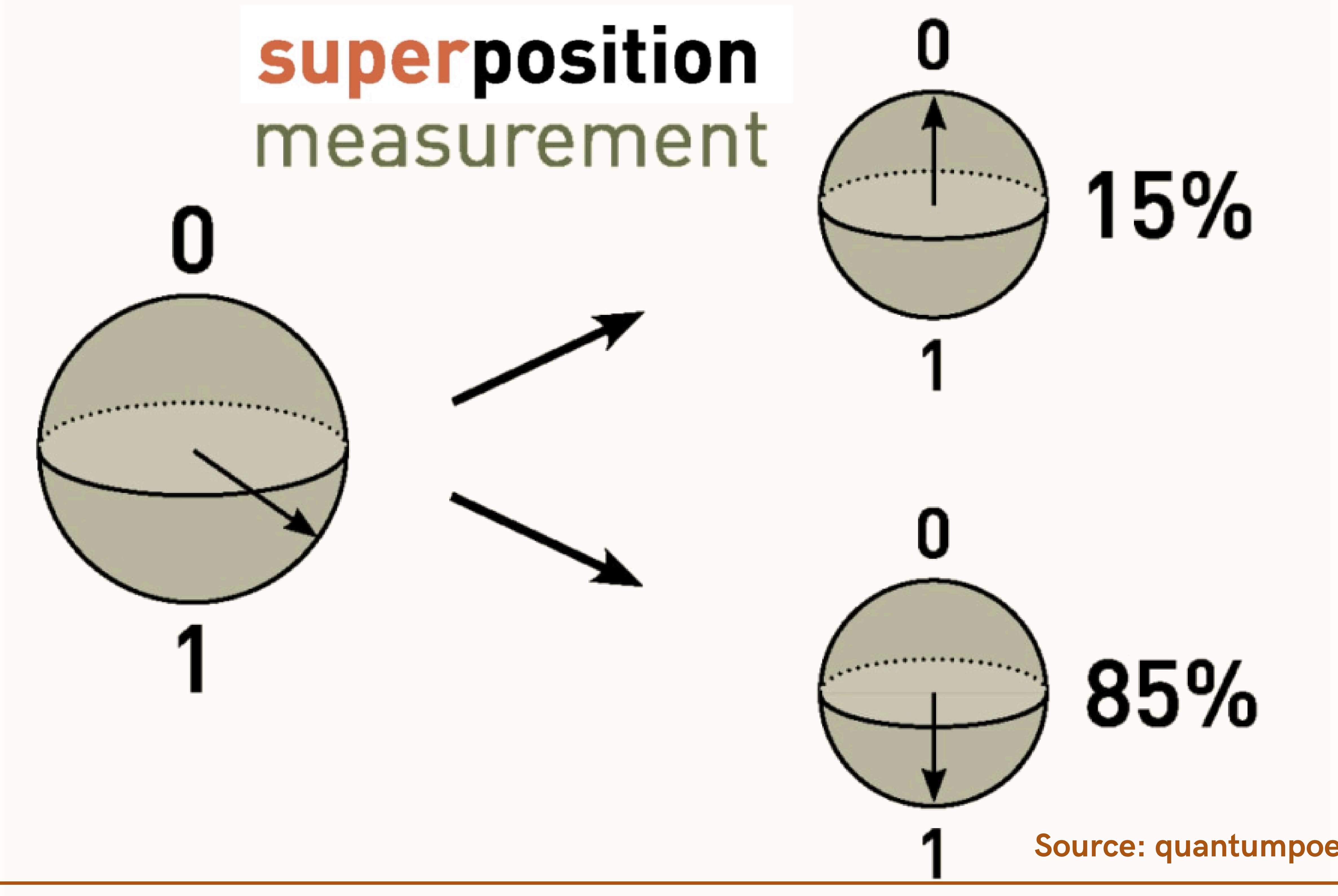
1



Source: quantumpoet.com

KEY CONCEPTS IN QUANTUM COMPUTING (1/4)

QUBITS AND SUPERPOSITION



Source: quantumpoet.com

1

KEY CONCEPTS IN QUANTUM COMPUTING (2/4)

ENTANGLEMENT

With **2 qubits**, the global state lives in a **4-dimensional space**.

Some states cannot be written as “state of qubit A × state of qubit B”: these are **entangled states.**

Example (Bell state): Equal superposition of $|00\rangle$ and $|11\rangle$

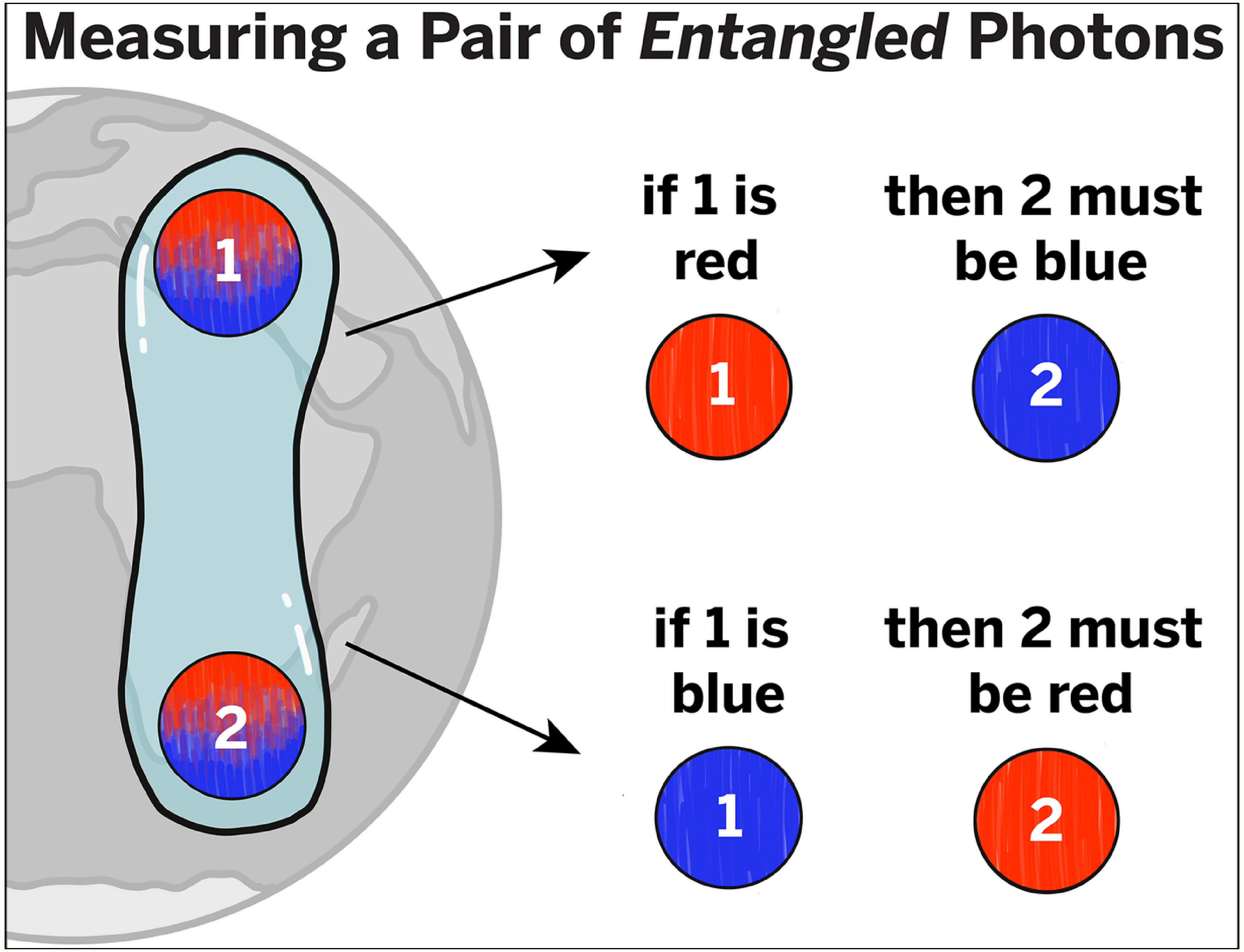
Measurements are **perfectly correlated**:

> if Alice gets 0, Bob always gets 0;
>
> if Alice gets 1, Bob always gets 1.

1

KEY CONCEPTS IN QUANTUM COMPUTING (2/4)

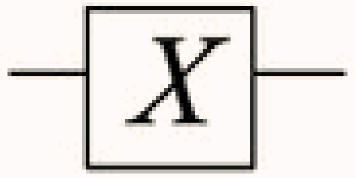
ENTANGLEMENT



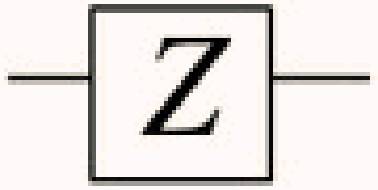
Source: the Quantum Atlas

KEY CONCEPTS IN QUANTUM COMPUTING (3/4)

ELEMENTARY GATES



$$|0\rangle \longrightarrow |1\rangle$$
$$|1\rangle \longrightarrow |0\rangle$$



$$|0\rangle \longrightarrow |0\rangle$$
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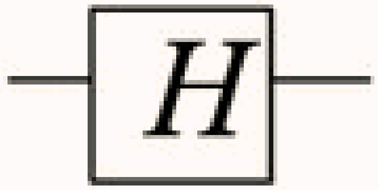
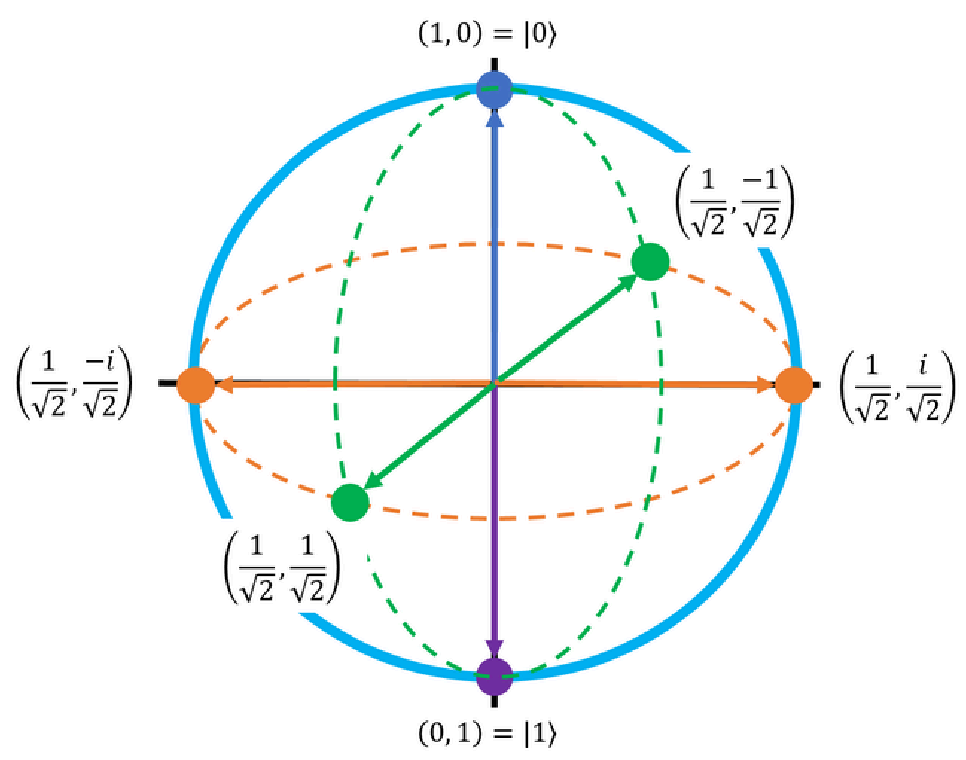


$$|0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$|1\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$



KEY CONCEPTS IN QUANTUM COMPUTING (3/4)

ELEMENTARY GATES

1

X

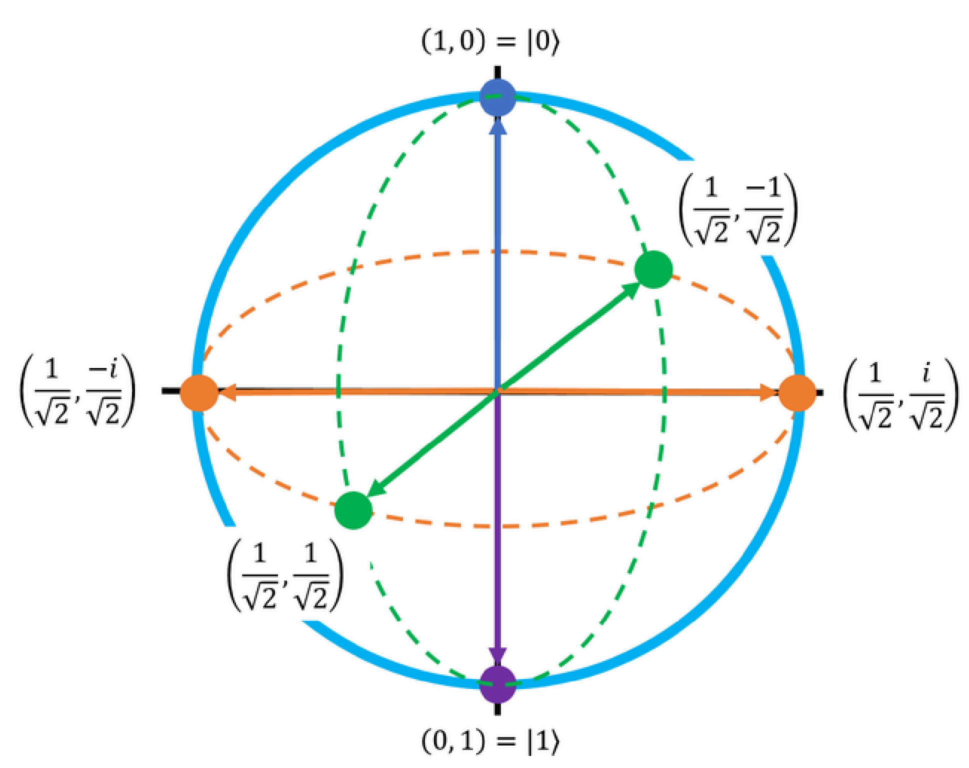
$$|0\rangle \longrightarrow |1\rangle$$
$$|1\rangle \longrightarrow |0\rangle$$

Z

$$|0\rangle \longrightarrow |0\rangle$$
$$|1\rangle \longrightarrow -|1\rangle$$

H

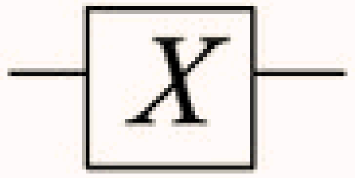
$$|0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$|1\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$



$$R_\phi = \begin{array}{l} |0\rangle \longrightarrow |0\rangle \\ |1\rangle \longrightarrow e^{i\phi}|1\rangle \end{array}$$

KEY CONCEPTS IN QUANTUM COMPUTING (3/4)

ELEMENTARY GATES



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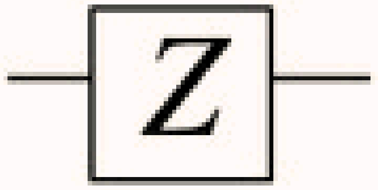

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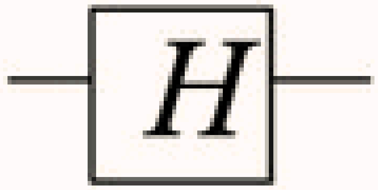
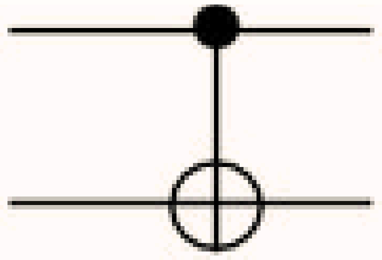
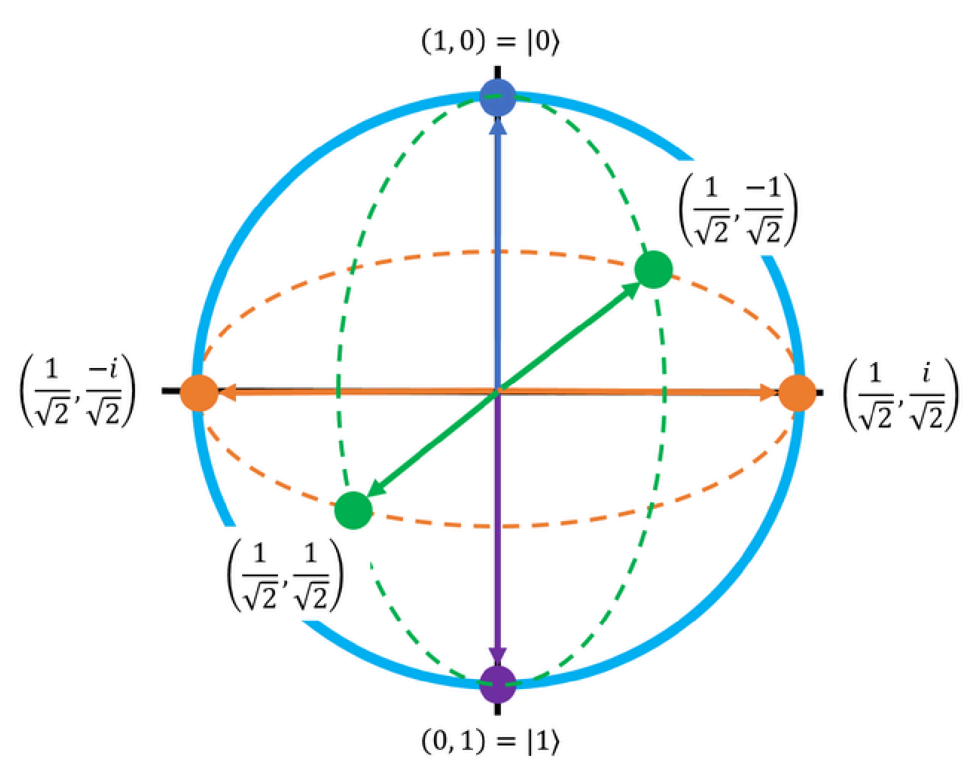


$$|0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$|1\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$



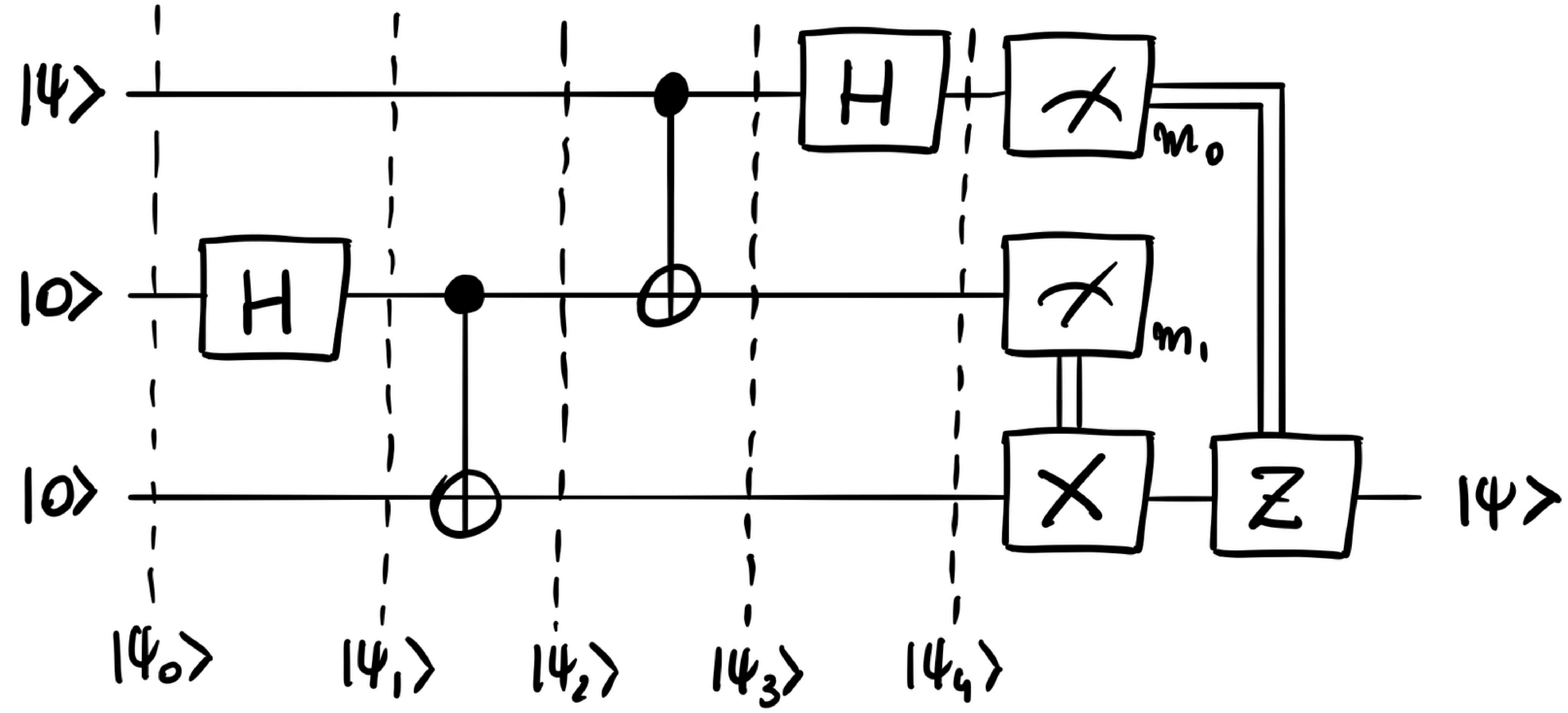
$$|a, b\rangle \longrightarrow |a, a+b\rangle$$



$$R_\phi = \quad |0\rangle \longrightarrow |0\rangle, \quad |1\rangle \longrightarrow e^{i\phi}|1\rangle$$

1 KEY CONCEPTS IN QUANTUM COMPUTING (4/4)

STANDARD QUANTUM CIRCUITS



Source: TeXample

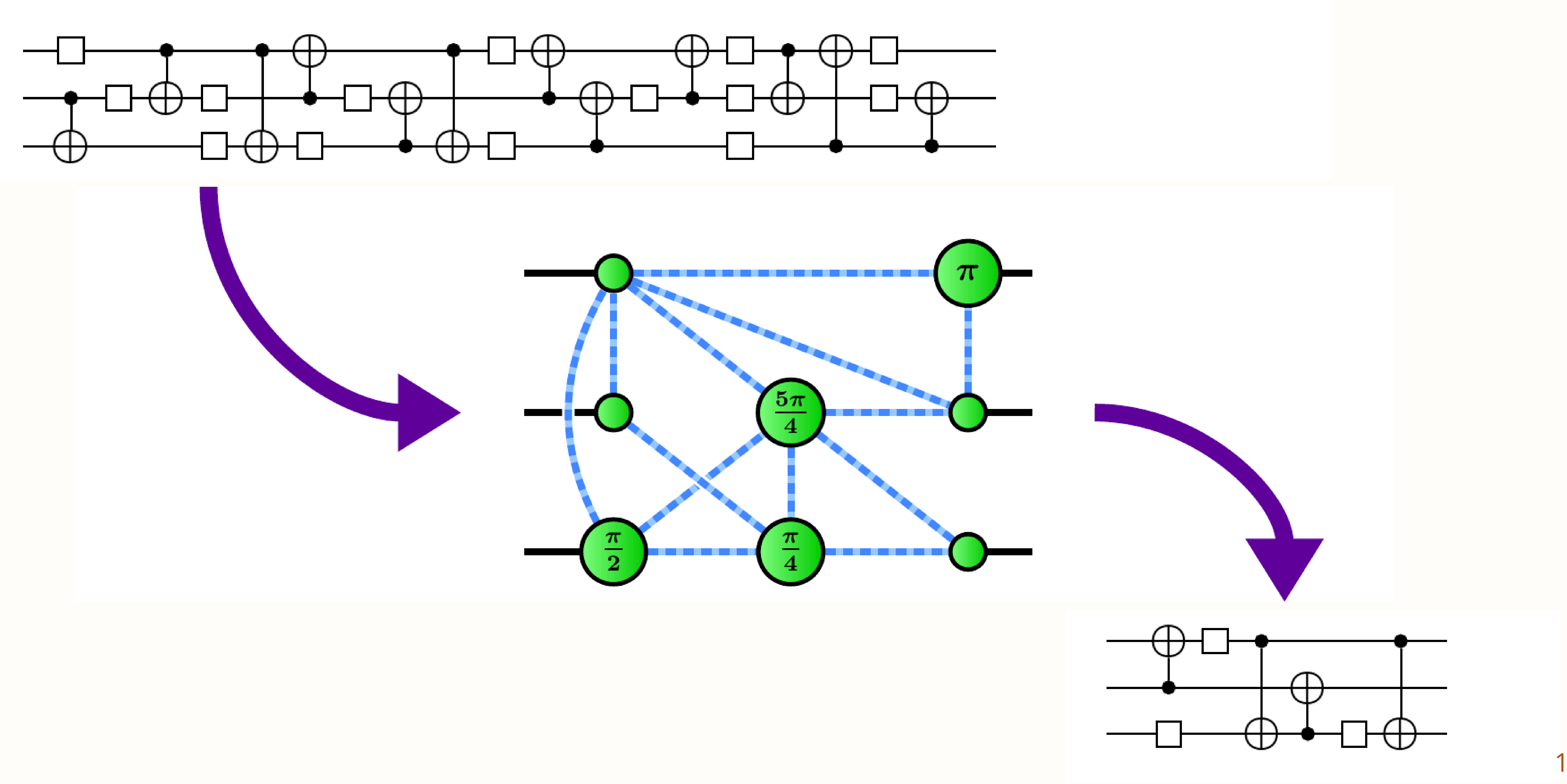
1

Kahoot!

2

FROM CIRCUITS TO GRAPHS (1/3)

WHY LOOK BEYOND CIRCUIT DIAGRAMS?



Source: Perdrix & al.

2

FROM CIRCUITS TO GRAPHS (2/3)

QUANTUM CIRCUITS → GRAPH STATES

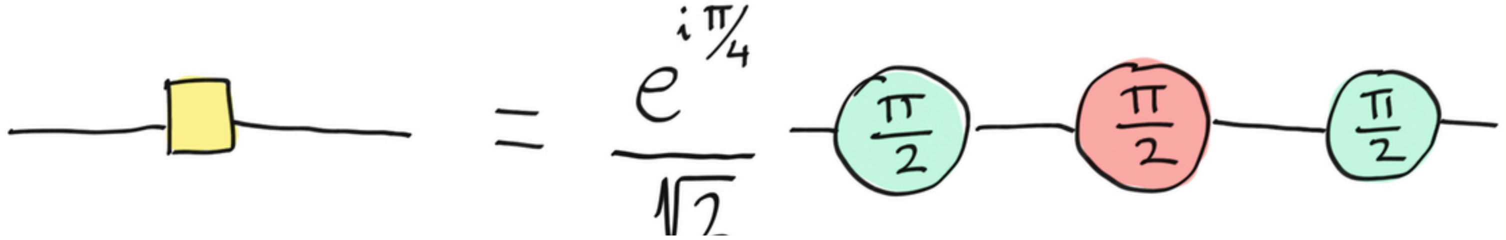
Computation then proceeds by measuring vertices in suitable bases.
This already suggests : **quantum processes can be encoded as graphs + labels.**

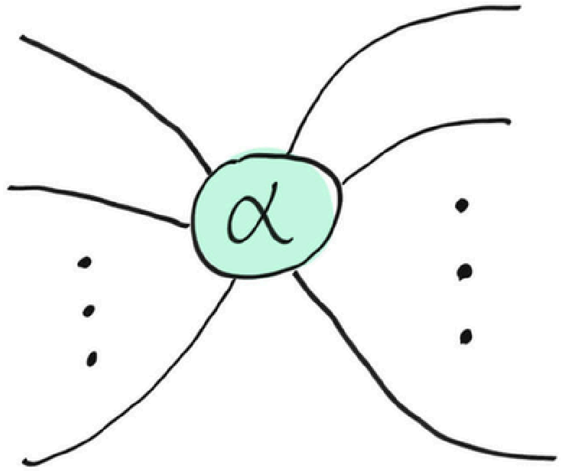
The ZX-calculus generalises this idea: **every circuit can be translated into a labeled graph with special nodes**.

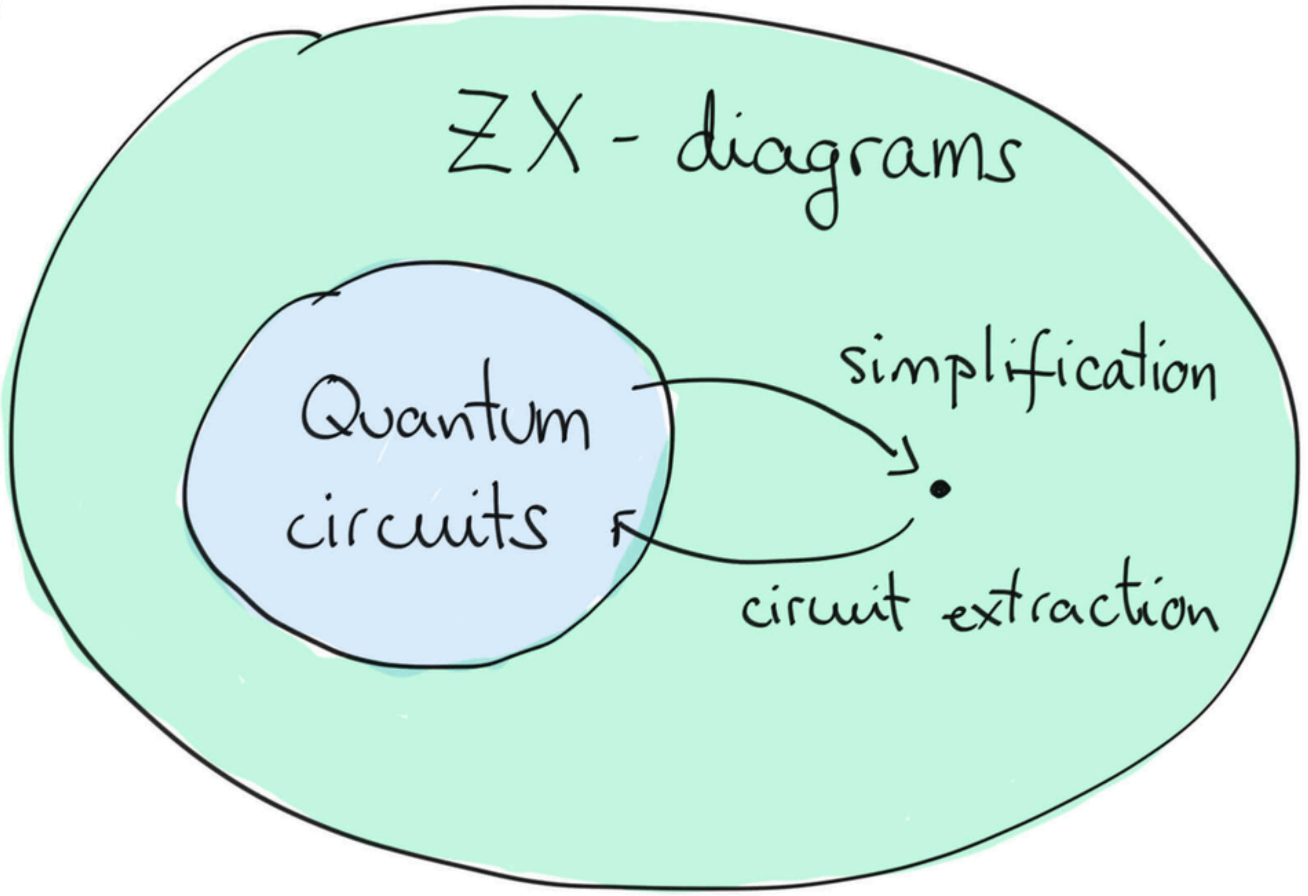
2

FROM CIRCUITS TO GRAPHS (3/3)

THE ZX-CALCULUS



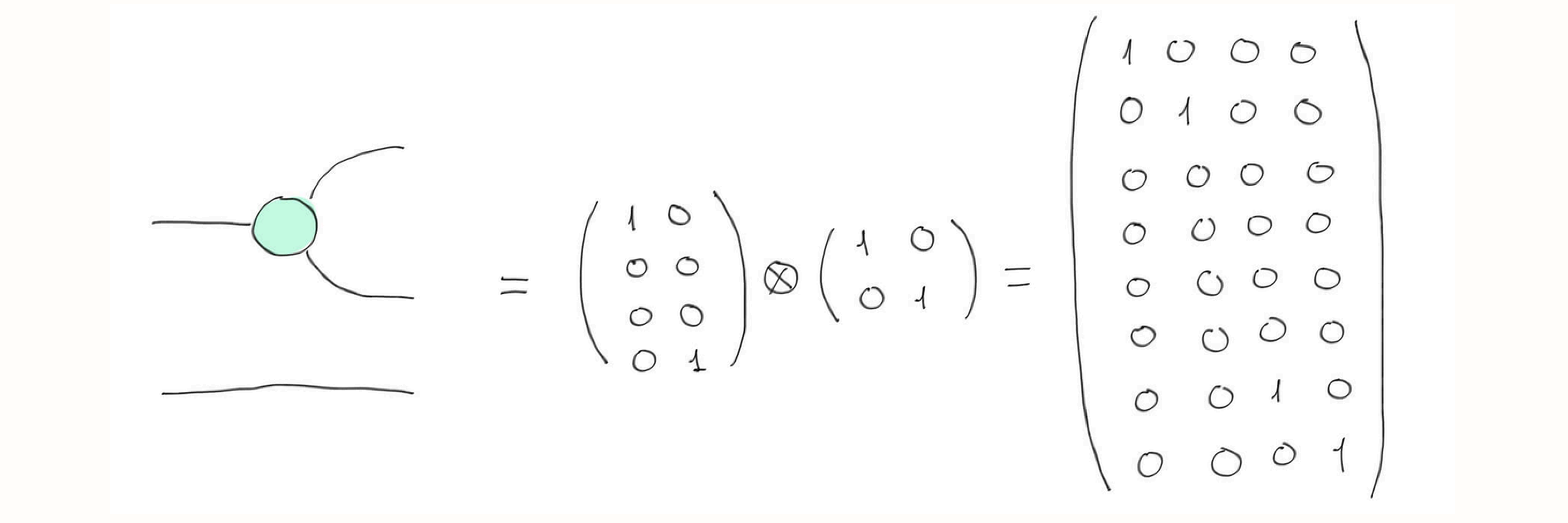


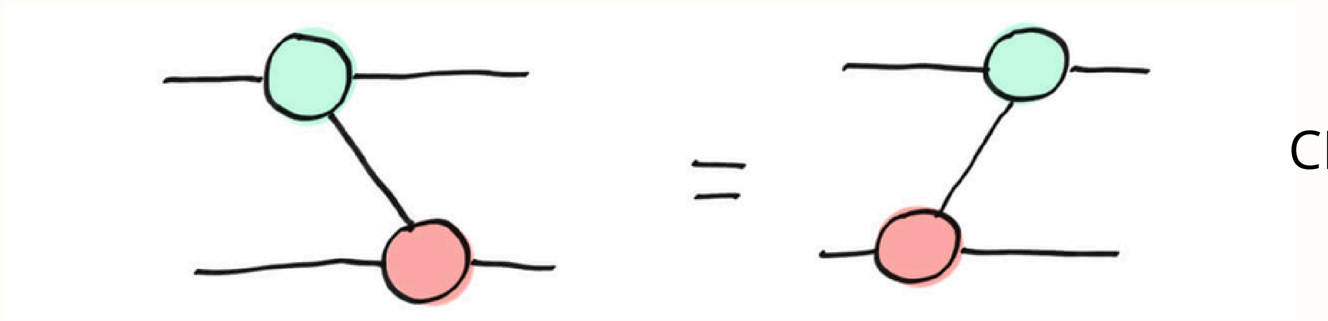


Source: PennyLane

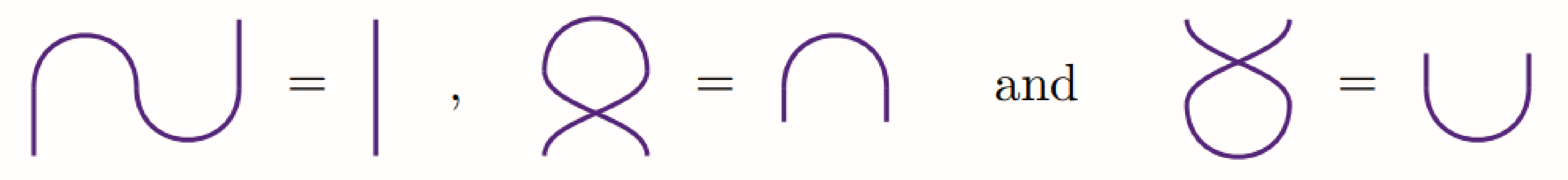
2 FROM CIRCUITS TO GRAPHS (3/3)

THE ZX-CALCULUS





CNOT



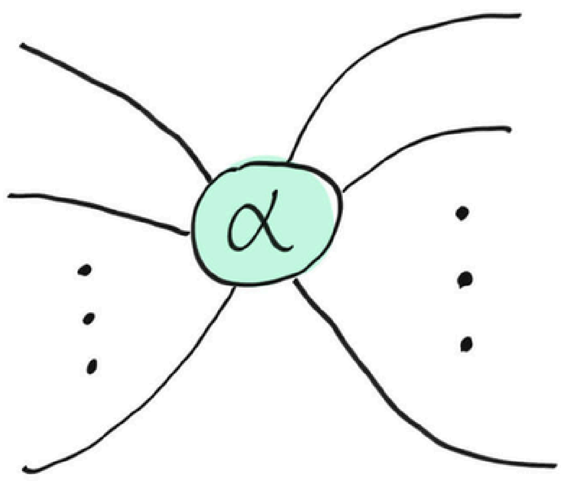
Source: PennyLane

Source: PennyLane

2

Kahoot!

ZX-CALCULUS : WHAT ARE SPIDERS?

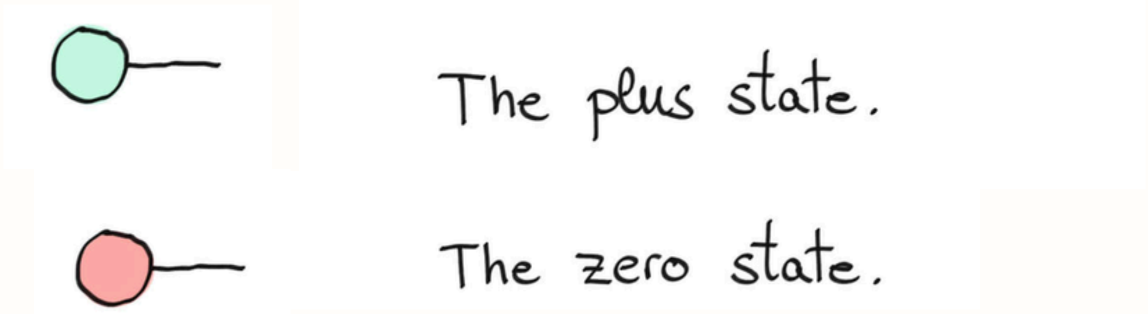


- Inputs/outputs are just wires (qubits)
- Enforces all wires carry the same bit (in Z basis)

$$|0000\rangle \to |000\rangle$$
$$|1111\rangle \to e^{i\alpha}|111\rangle$$

Green : Copy/compare in the Z basis

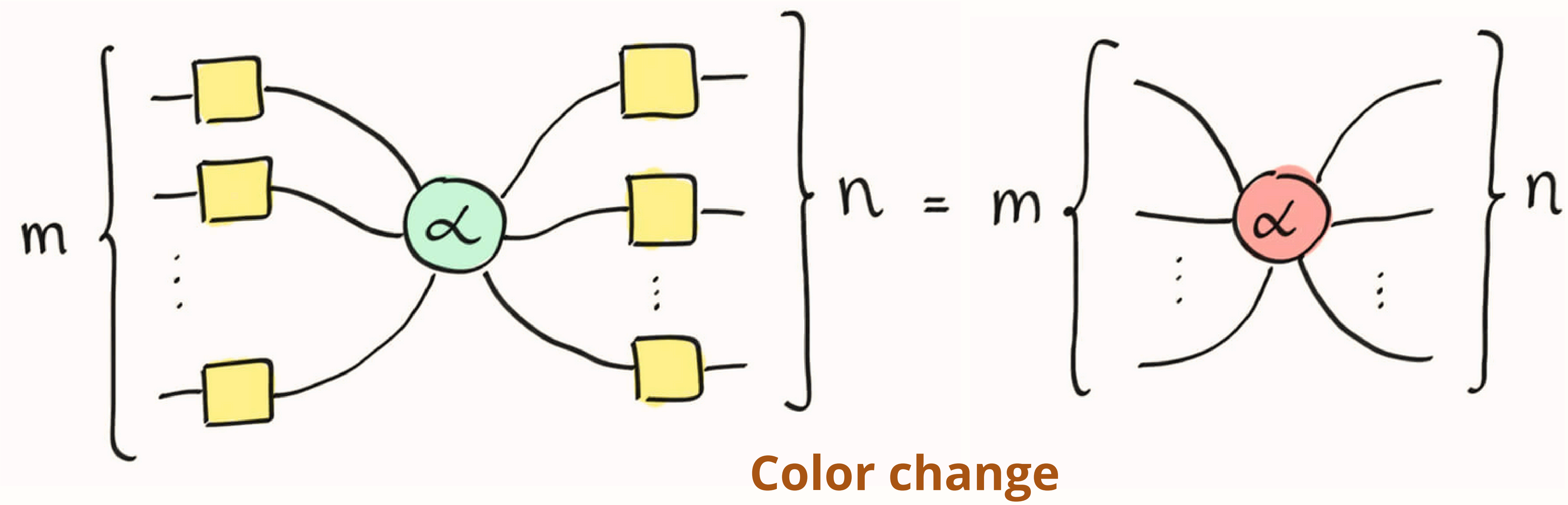
Red : Copy/compare in the X basis



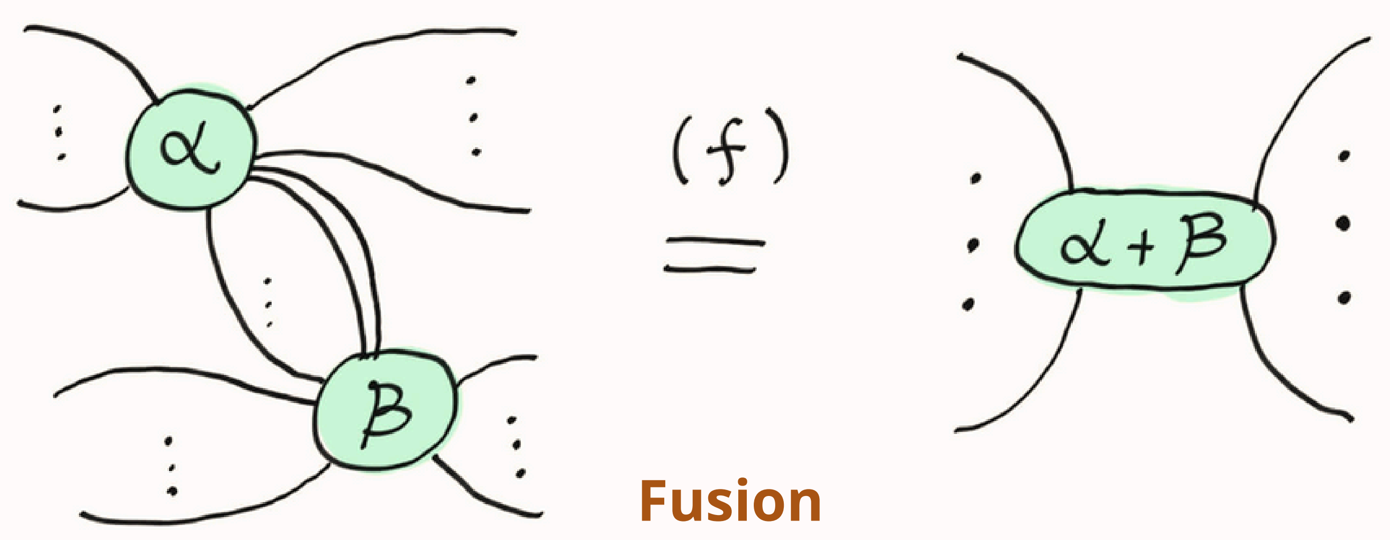
All the rewrite rules are just identities between the linear maps

3

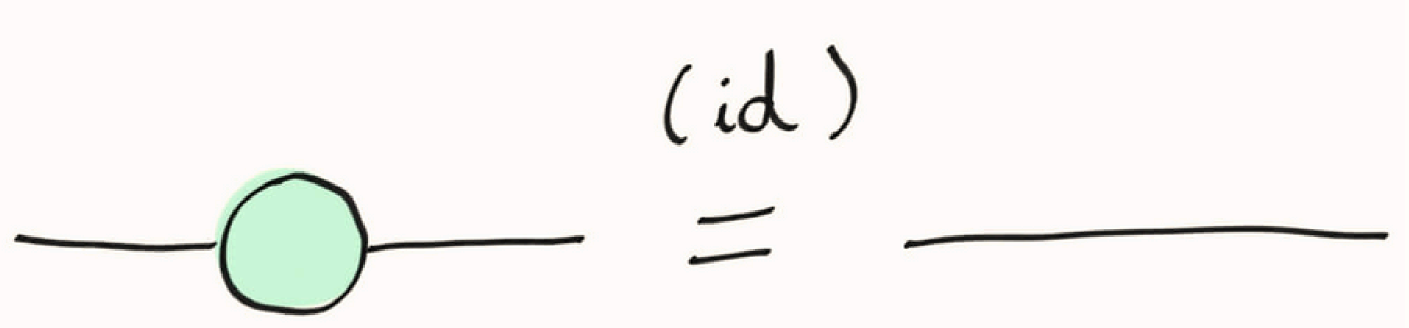
ZX-CALCULUS REWRITE RULES (1/2)
FUSION, IDENTITY, COLOUR CHANGE



Color change



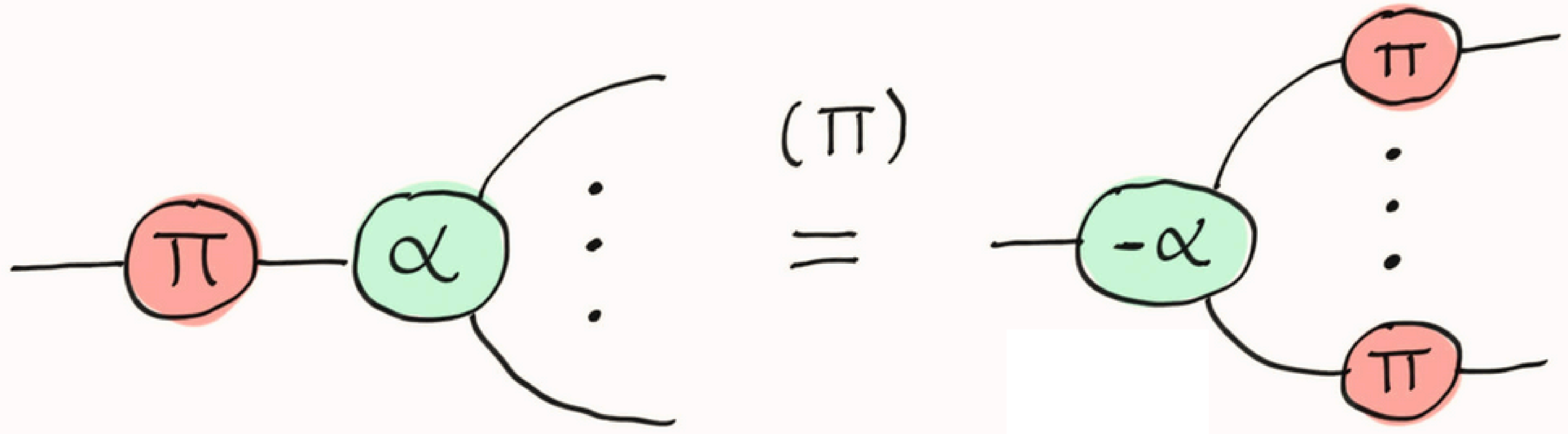
Fusion



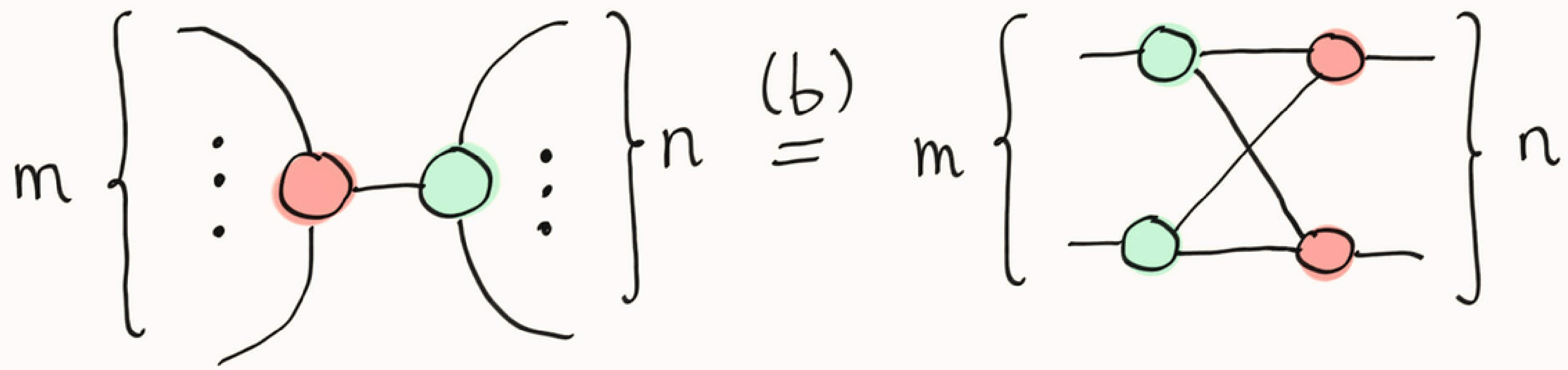
Identity

Source: PennyLane

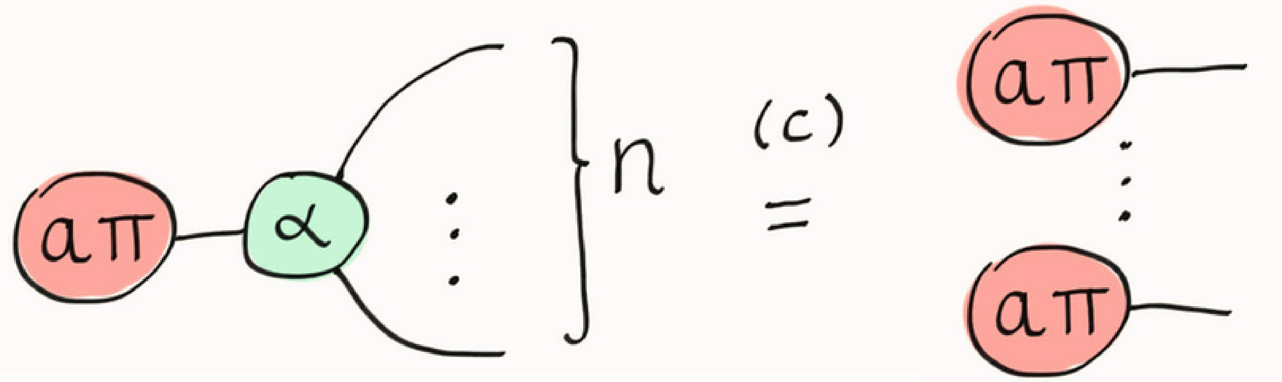
ZX-CALCULUS REWRITE RULES (2/2)
INTERACTION RULES & HOPF STRUCTURE



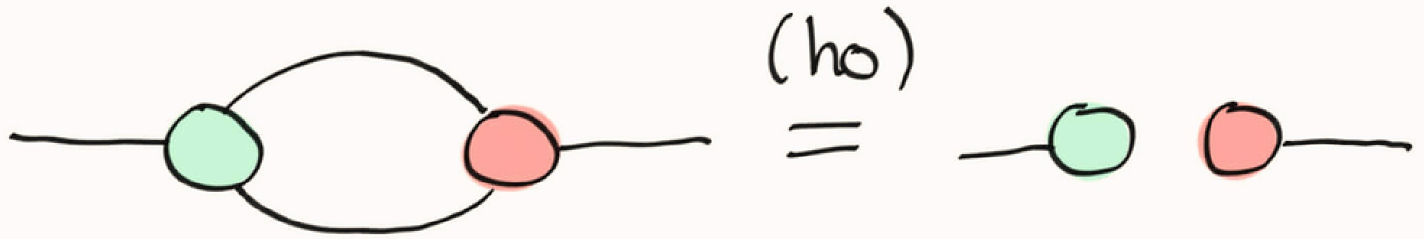
PI-copy



Bialgebra



state copy



Hopf

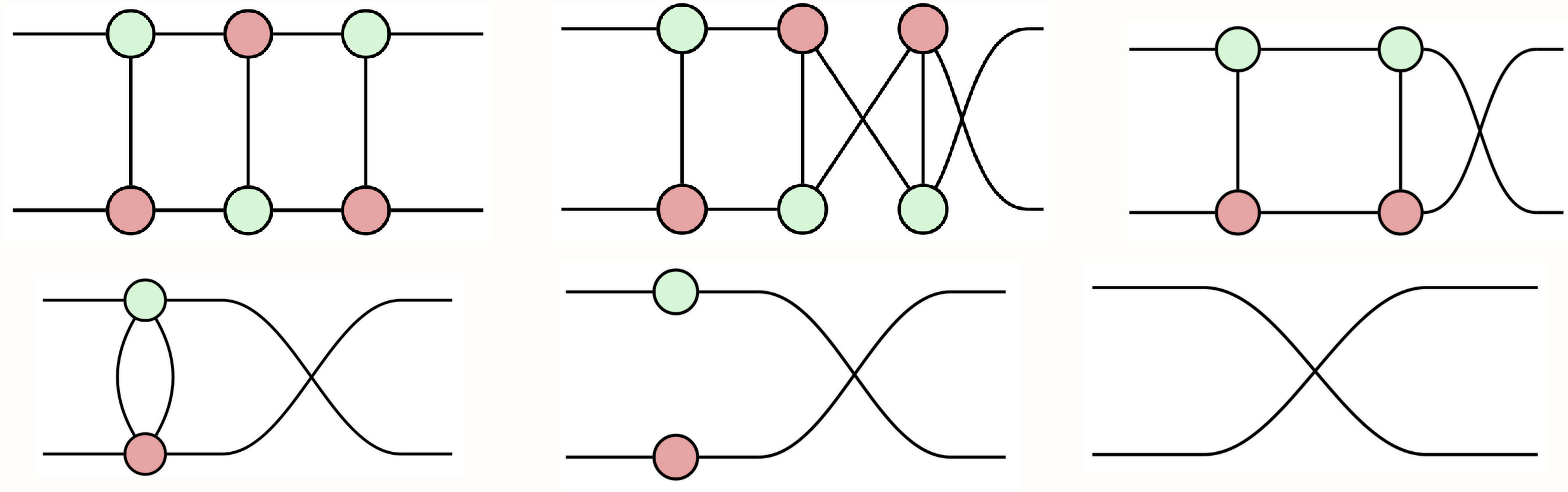
Source: PennyLane

3

Kahoot!

EXAMPLES OF SIMPLIFICATIONS (1/3)

SWAP



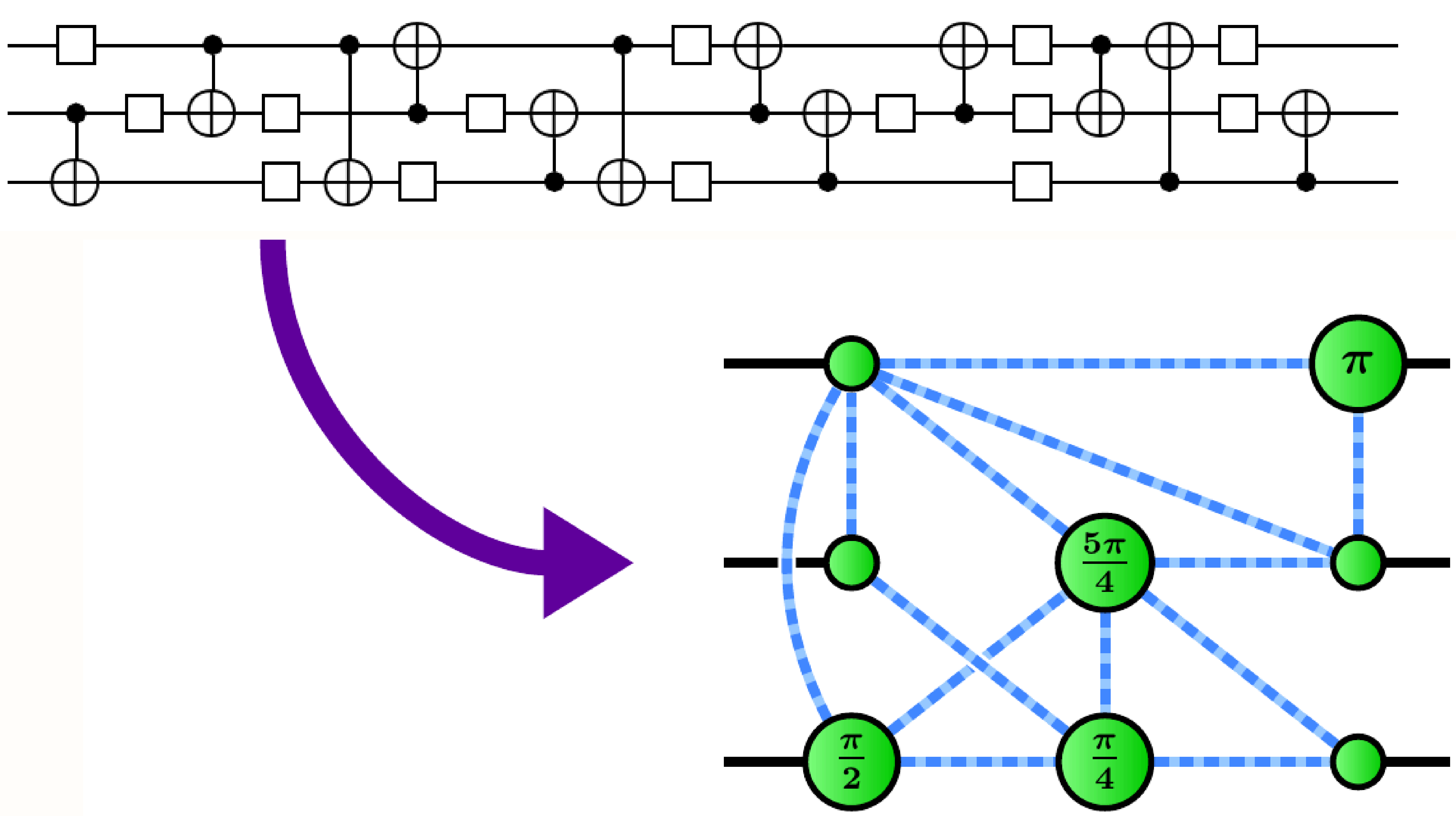
Source: ZX-calculus website

4

EXAMPLES OF SIMPLIFICATIONS (2/3)

GRAPH STATES, LOCAL COMPLEMENTATION, ...

1
2
3
4
5
6
7

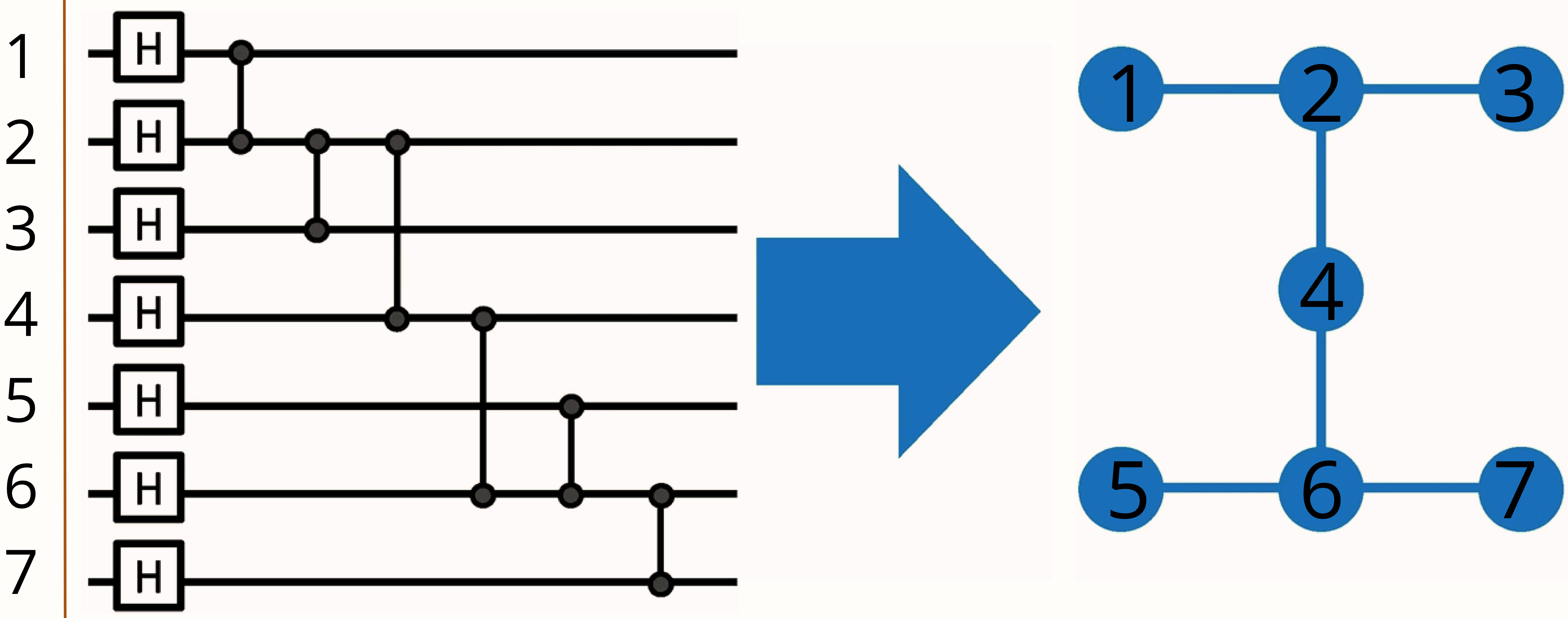


Source: Brandhofer & al.

EXAMPLES OF SIMPLIFICATIONS (2/3)

GRAPH STATES, LOCAL COMPLEMENTATION, ...

4

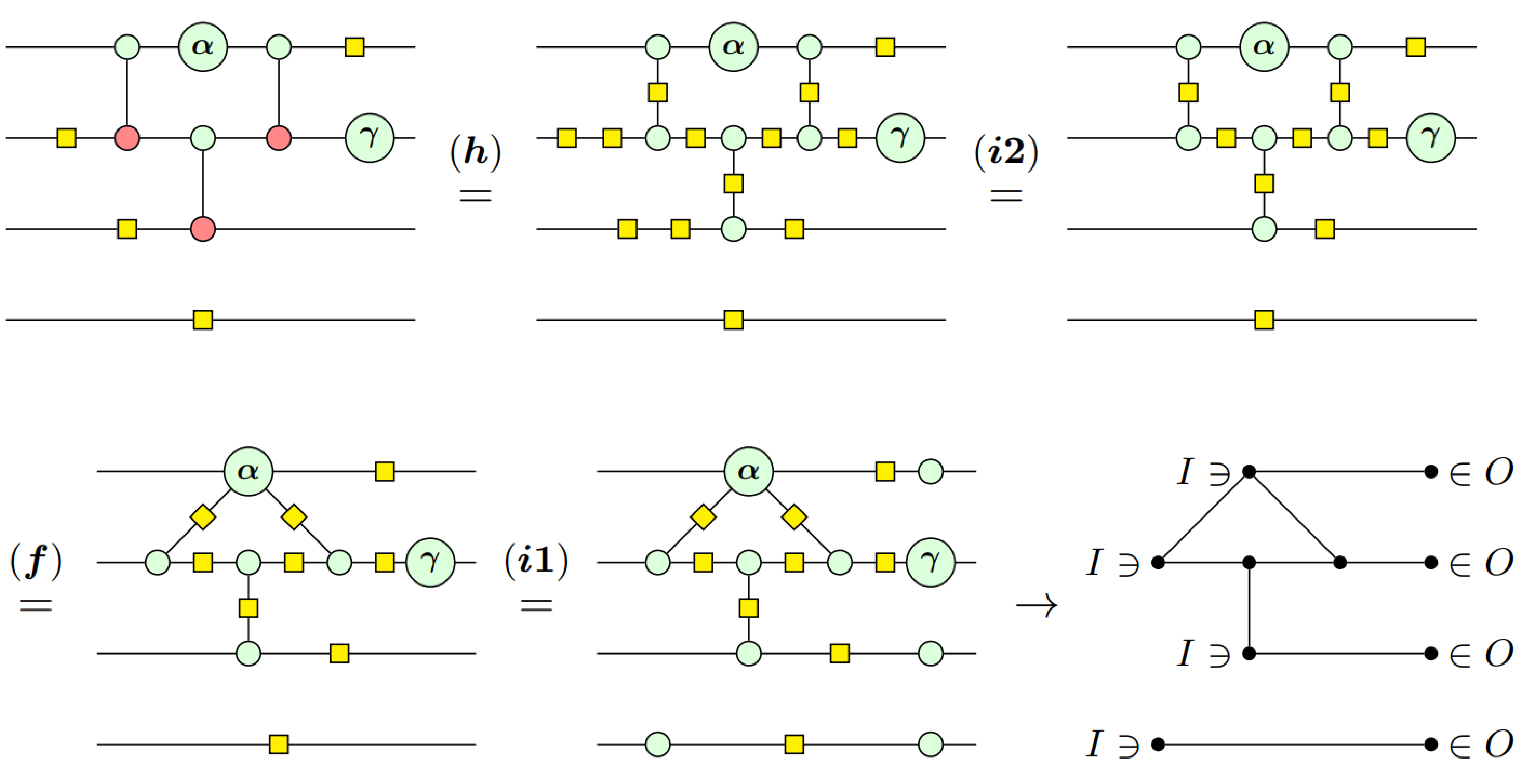


Source: Brandhofer & al.

EXAMPLES OF SIMPLIFICATIONS (2/3)

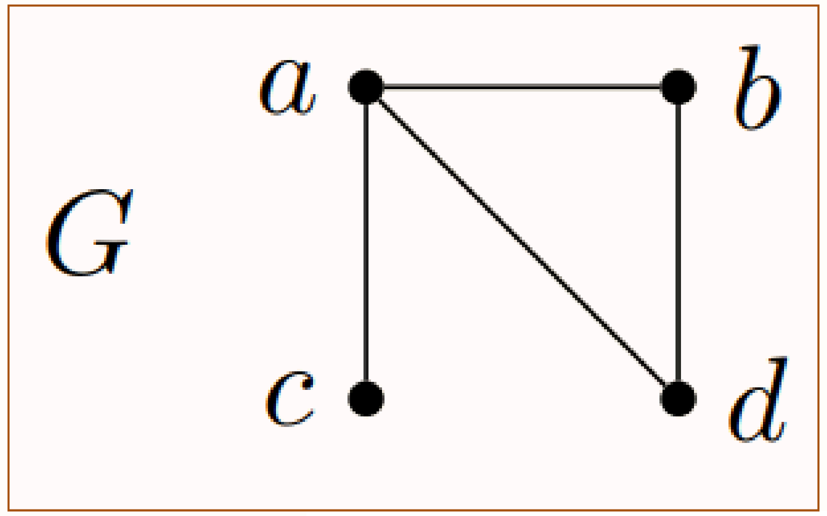
GRAPH STATES, LOCAL COMPLEMENTATION, ...

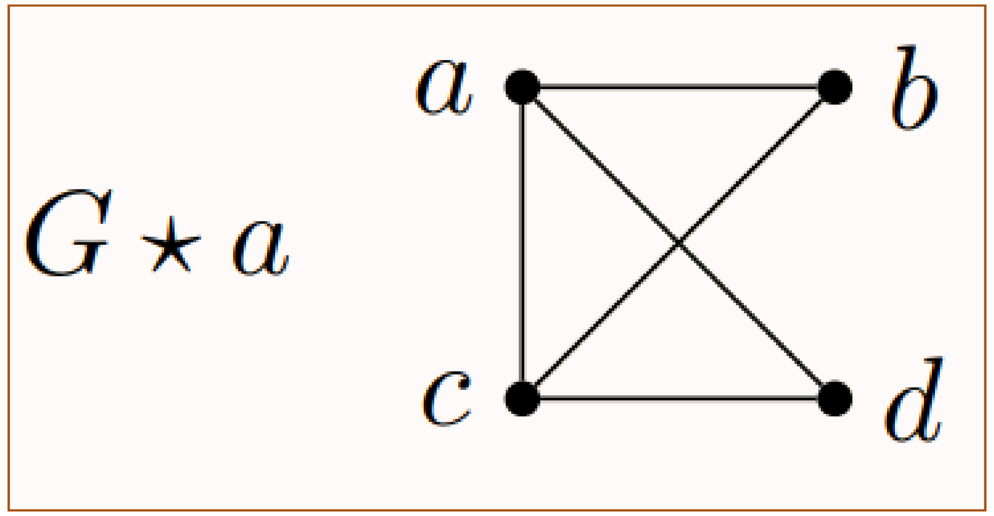
4

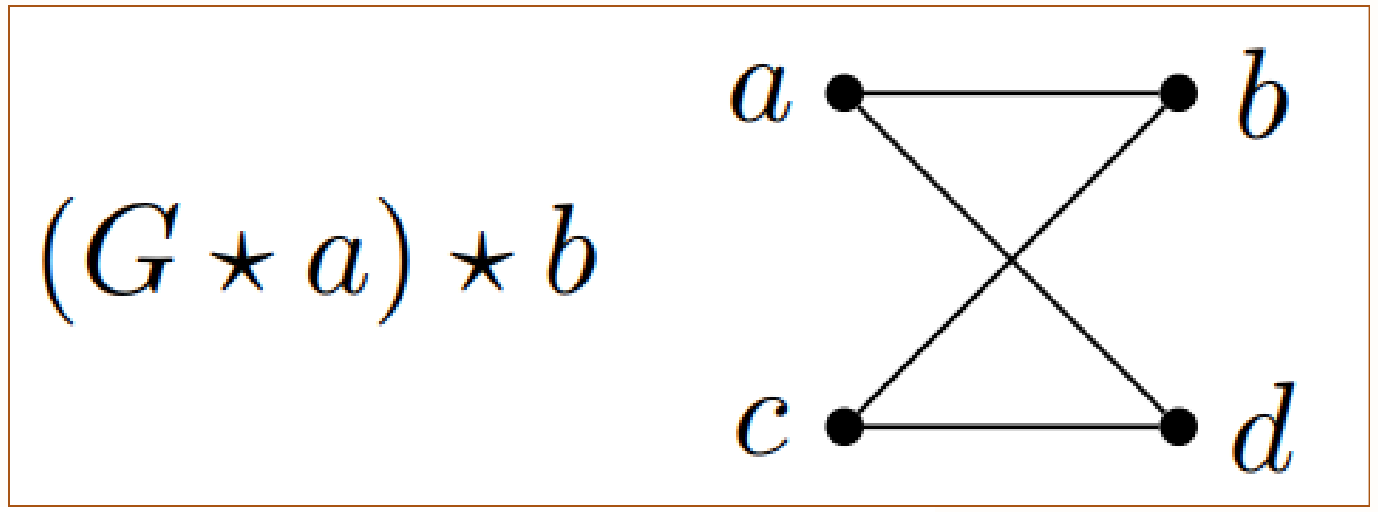


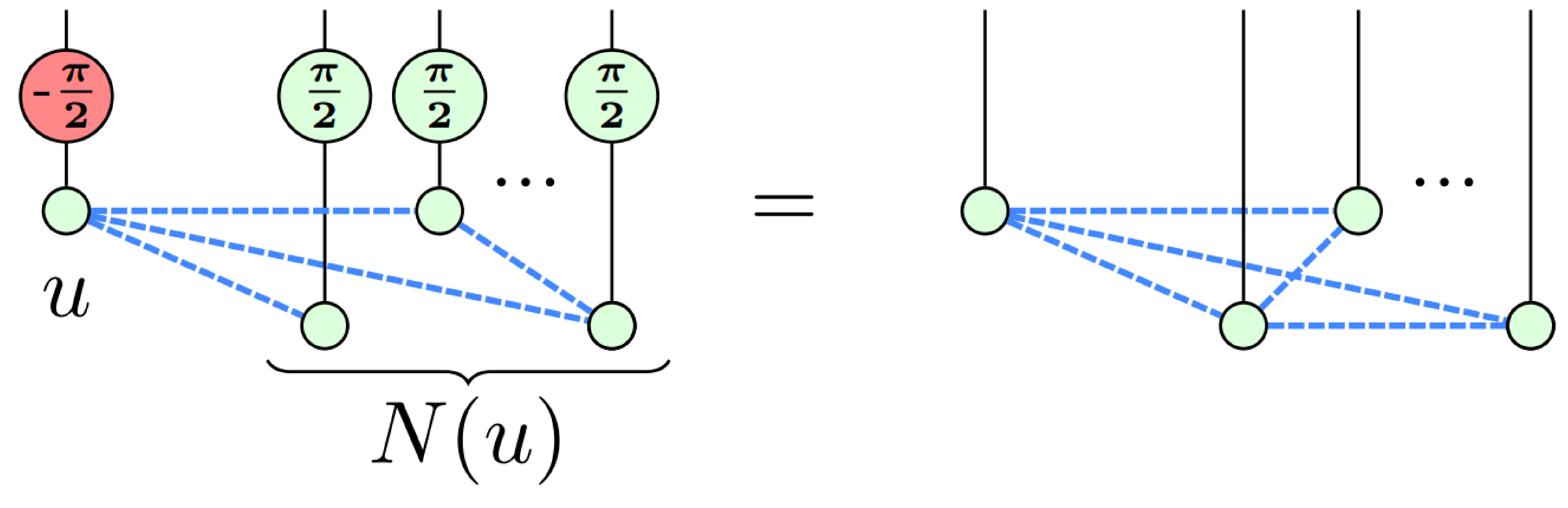
EXAMPLES OF SIMPLIFICATIONS (2/3)

GRAPH STATES, LOCAL COMPLEMENTATION, ...



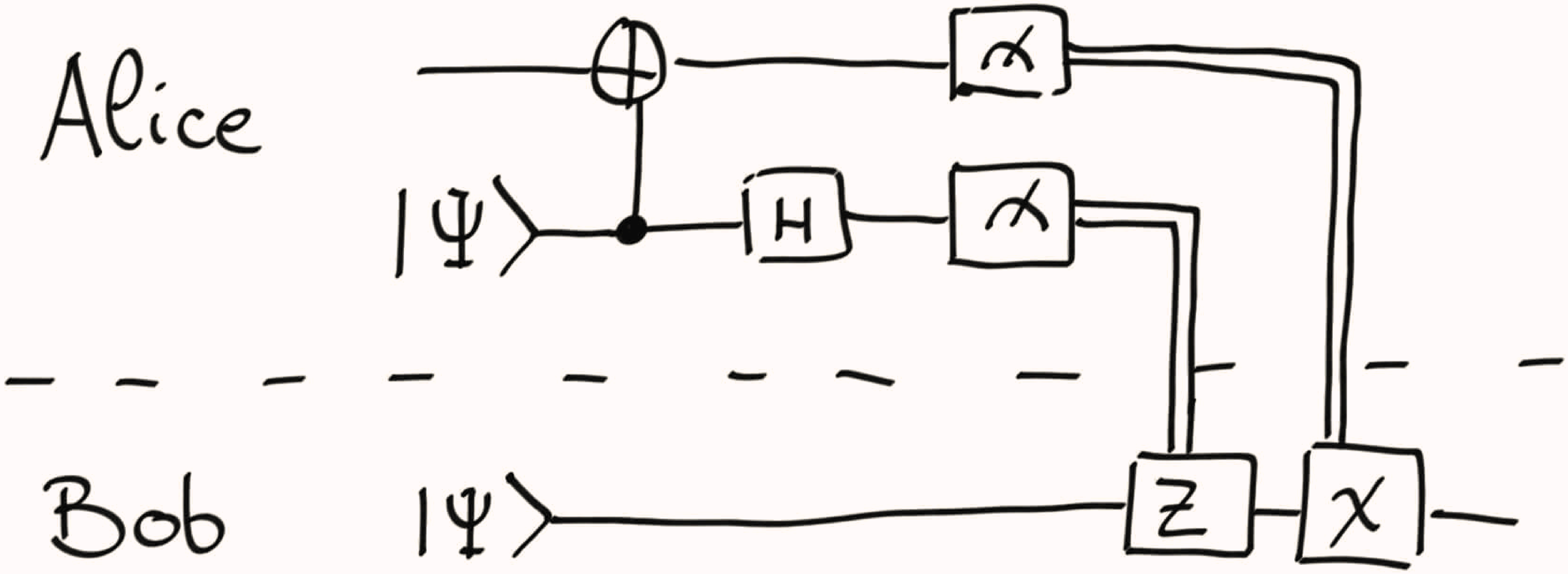






EXAMPLES OF SIMPLIFICATIONS (3/3)

QUANTUM TELEPORTATION IN ZX

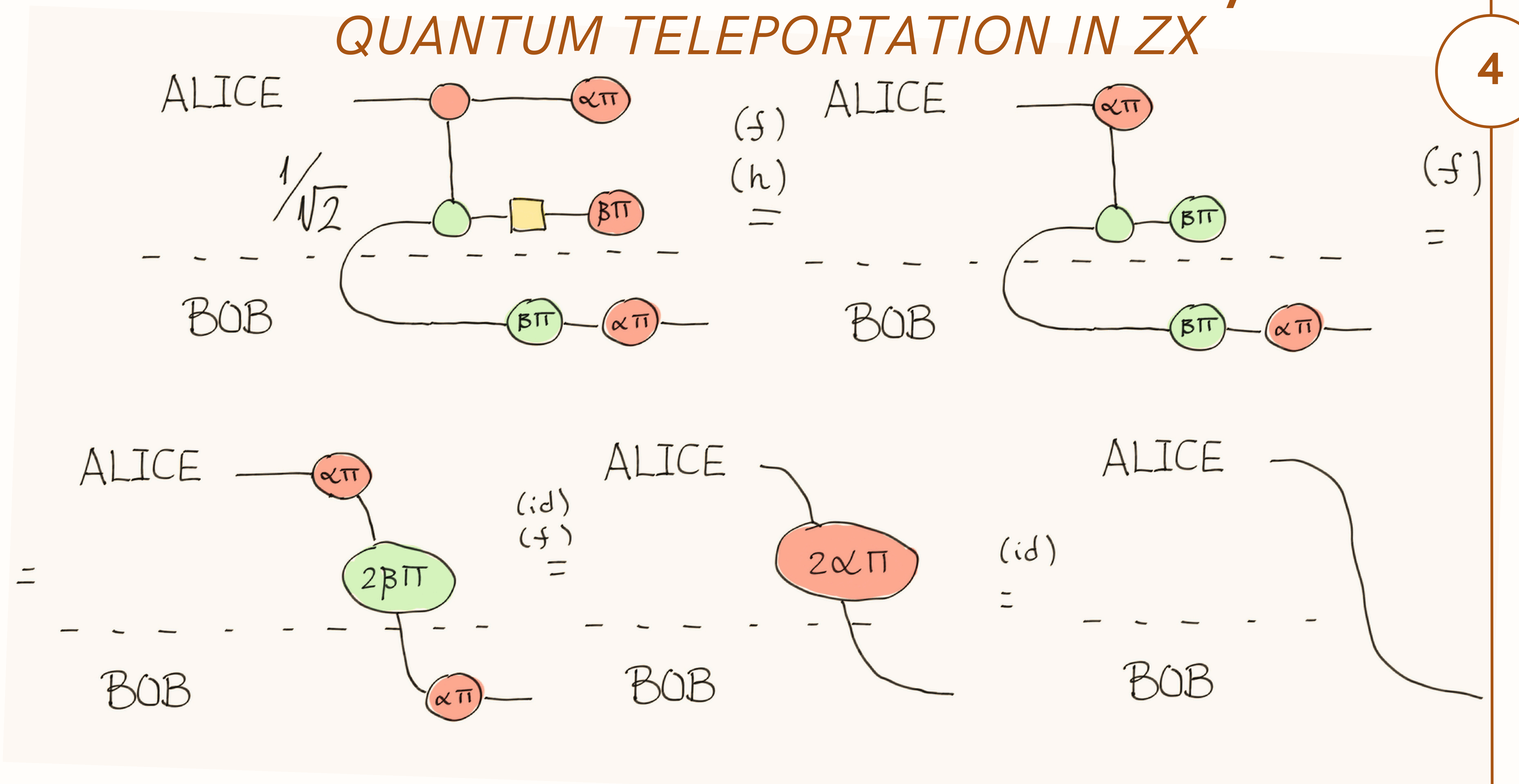


Source: PennyLane

EXAMPLES OF SIMPLIFICATIONS (3/3)

QUANTUM TELEPORTATION IN ZX

4



Source: PennyLane

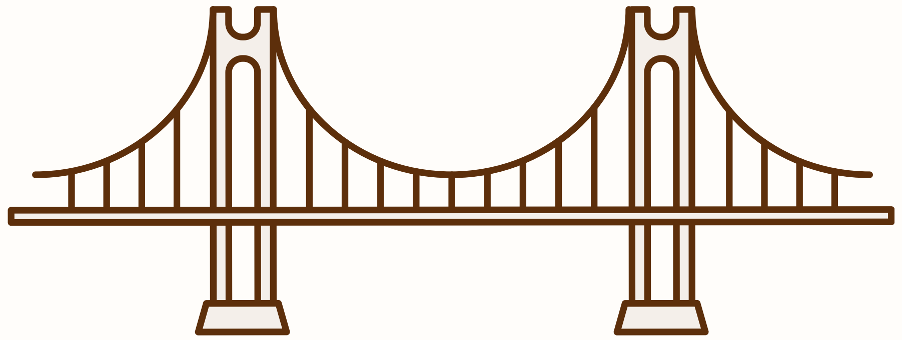
5

DEMONSTRATION ZX-LIVE

OPEN PROBLEMS & MY PHD TOPIC

Qubits (d = 2)

✅ Stabilizer
✅ Clifford+T (certain fragments)
✅ Full pure qubit QM (ZX complete)



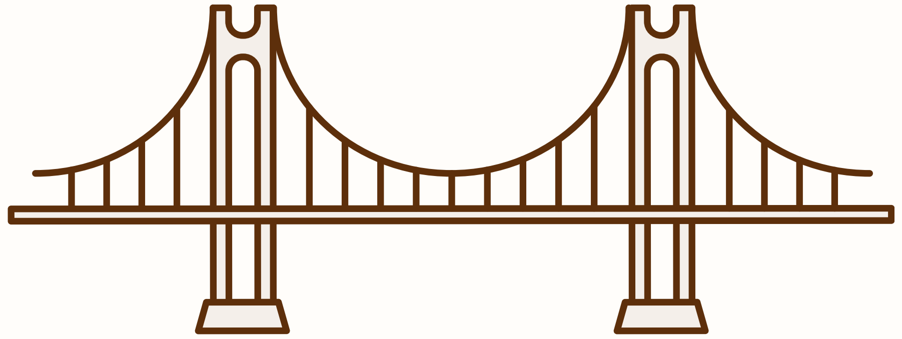
Qudits (d ≥ 3)

❓ ZX / ZW / ZH candidates
❓ Completeness in general
❓ Practical optimisation tools
❓ Basically everything

Even for Quantum Circuits…

Qubits (d = 2)

✅ Well-understood universal gate sets (e.g. Clifford+T)
✅ Rich theory of compilation & optimisation
✅ Many practical toolchains (Qiskit, tket, …)



Qudits (d ≥ 3)

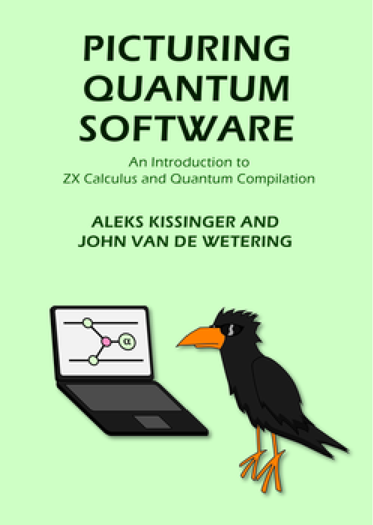
❓ Fewer standard gate sets and models
❓ No widely used equational theory
❓ Optimisation & verification much less developed
❓ Basically everything (again)

CONCLUSION – QUANTUM COMPUTING AS GRAPH REWRITING

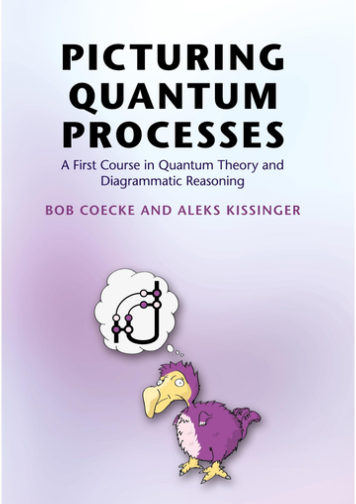
Behind the scary matrices, quantum computing can be seen as graphs plus rewrite rules.
The ZX-calculus is basically graph theory and category theory applied to quantum computing

ZX touches:
combinatorics (graphs, Hopf algebras, Fuss-Catalan algebras...),
logic & automata (equational theories, completeness, decision problems),
algorithms & optimisation (graph rewriting, scheduling, constraint handling).

Questions, collaborations, or spider-related nightmares:
colin.blake@inria.fr



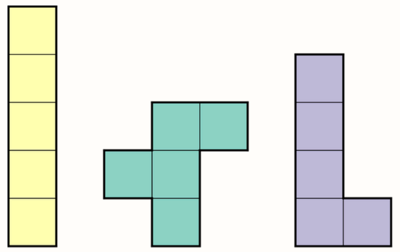
GITHUB.COM/ZXCALC/BOOK



PICTURING QUANTUM PROCESSES

ZXCALCULUS.COM

ZX-CALCULUS & LIGM: POSSIBLE CONNECTIONS

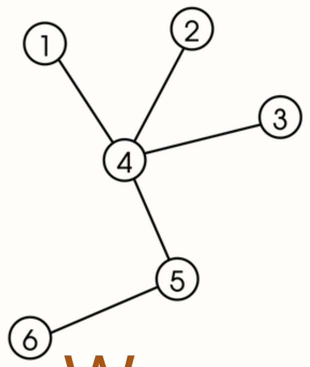


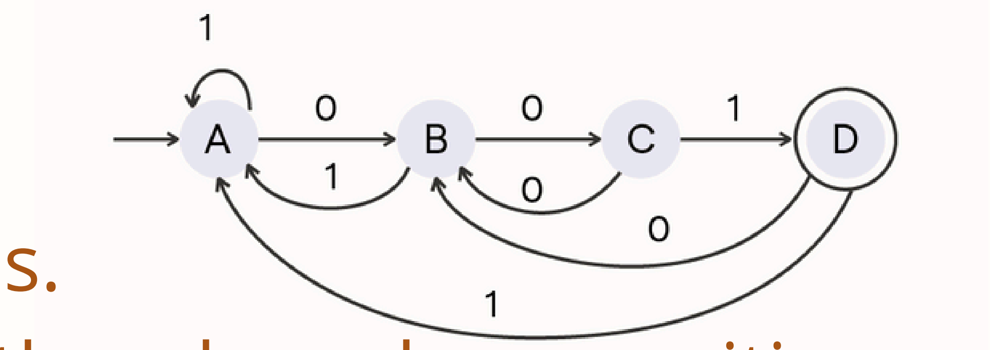
Algebraic & enumerative combinatorics:

ZX diagrams are labelled planar graphs with a rich algebraic structure.

Rewrite rules correspond to identities in a kind of combinatorial Hopf algebra on spiders/graphs.

Natural questions: normal forms, counting diagrams up to rewrites, random ZX diagrams, etc.

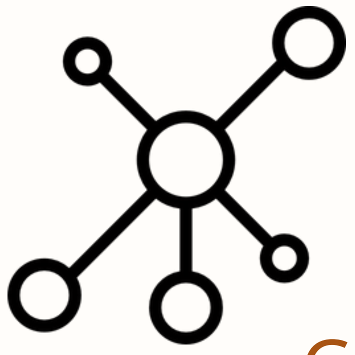




Logic, automata & models:

Equivalence of circuits ⇔ equivalence of ZX diagrams.

We can study the complexity of these decision problems and design algorithms based on rewriting and normal forms.

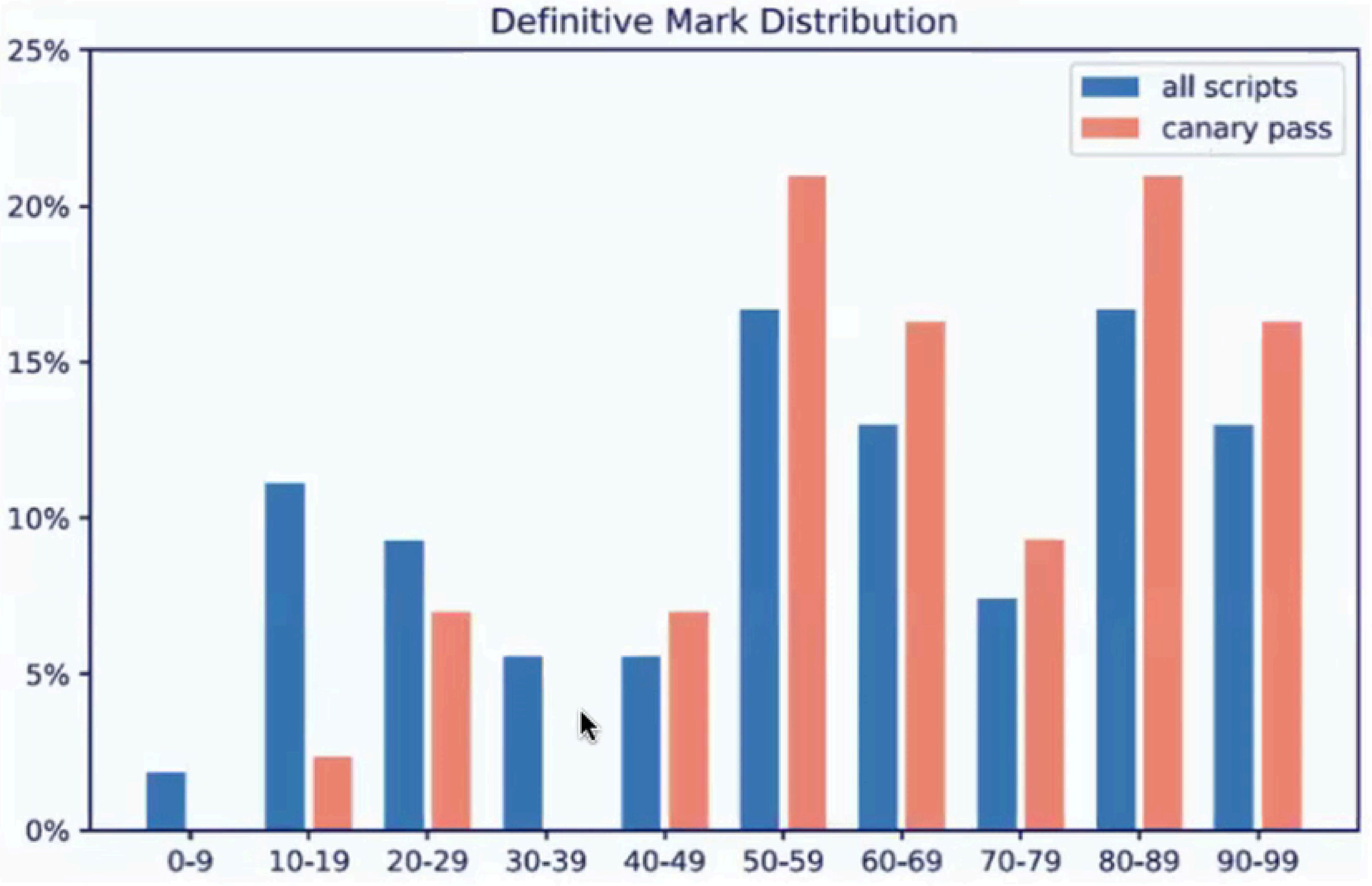


Algorithms, networks:

Circuit optimisation becomes a constrained graph rewriting / scheduling problem.

There is room for ideas from graph algorithms, constraint satisfaction, scheduling, and even geometric / topological methods.

Experiment results



Canary evaluation

- 82% pass rate
- 48% distinctions

Canary questions as a benchmark for assessment, ensuring understanding of **the most fundamental concepts** or skills.

Source: Lia Yeh's work

Liens utiles:

https://zxcalculus.com/ → Site de référence, avec des tutos visuels, notamment la dérivation de la quantum teleportation qui est bien

https://zxcalculus.com/intro.html

https://github.com/zxcalc/zxlive → Pour générer des circuits

https://www.cs.ox.ac.uk/people/bob.coecke/ZX-lectures_JPG.pdf → "Basic ZX-calculus for students and professionals"

https://arxiv.org/abs/2012.13966 → Un peu ancien bouquin de référence

https://quantuminpictures.org/resources/ → le cours complet de Quantum in Pictures, avec plein de slides à copier

https://www.reddit.com/r/TheoreticalPhysics/comments/mtyk3n/bob_coecke_from_quantum_processes_to_cognition/ → Une vidéo de Bob Coecke, il y a des slides qui introduisent le ZX-calcul donc c'est bien

https://www.quantinuum.com/blog/quantum-in-pictures

https://github.com/zxcalc/book → Le crow book, le bouquin le plus moderne

https://pennylane.ai/qml/demos/tutorial_zx_calculus

https://www.youtube.com/watch?v=J3Jh7Cej6og → ZX calculus and education, ça peut être quelque chose de sympa à mettre en conclusion 2312.03653